FIITJEE **COMMON TEST**

PHYSICS, CHEMISTRY & MATHEMATICS

CPT2 - 2 | **CODE : SET-A** | **PAPER - 2**

Time Allotted: 3 Hours **Maximum Marks: 312**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.
- You are not allowed to leave the Examination Hall before the end of the test.

BATCHES–Two Year CRP (1820) B LOT

INSTRUCTIONS

Caution: Question Paper CODE as given above MUST be correctly marked in the answer OMR sheet before attempting the paper. Wrong CODE or no CODE will give wrong results.

A. General Instructions

1. Attempt ALL the questions. Answers have to be marked on the OMR sheets.
2. This question paper contains **Three Sections.**
3. **Section-I** is Physics, **Section-II** is Chemistry and **Section-III** is Mathematics.
4. Each part is further divided into **Two Parts**: **Part A & B**
5. Rough spaces are provided for rough work inside the question paper. No additional sheets will be provided for rough work.
6. Blank Papers, clip boards, log tables, slide rule, calculator, cellular phones, pagers and electronic devices, in any form, are not allowed.

B. Filling of OMR Sheet

1. Ensure matching of OMR sheet with the Question paper before you start marking your answers on OMR sheet.
2. On the OMR sheet, darken the appropriate bubble with ***Blue/Black Ball Point Pen*** for each character of your Enrolment No. and write in ink your Name, Test Centre and other details at the designated places.
3. OMR sheet contains alphabets, numerals & special characters for marking answers.

C. Marking Scheme For All Three Parts.

(i) **Part-A (01 – 08)** contains 8 Multiple Choice Questions which have **Only One Correct answer.** Each question carries **+4 marks** for correct answer and **–2 marks** for wrong answer.

Part-A (09 – 16) contains **4 Paragraphs**. Based upon each paragraph, 2 Multiple Choice Questions have to be answered. Each question has Only One Correct answer and carries **+3 marks** for the correct answer and **–1 mark** for a wrong answer.

(ii) **Part-B (01 – 04)** contains **4 Matrix Match** Type Question which have statements given in 2 columns. Statements in the first column have to be matched with statements in the second column. There may be **One or More Than One Correct choices.** Each question carries **+12 marks** for all correct answer however for each correct row **+3 marks** will be awarded. There is no negative marking.

Name of the Candidate :_____

Batch :_____ **Date of Examination :**_____

Enrolment Number :_____

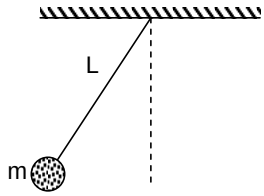
SECTION-I : PHYSICS

PART– A

(Single Correct Choice Type)

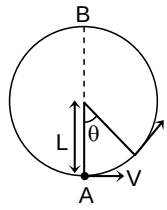
This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is:



(A) 9 (B) 18
(C) 27 (D) 36

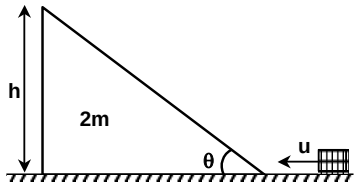
2. A bob of mass M is suspended by a massless string of length L. The horizontal velocity V at position A is just sufficient to make it reach the point B. The angle θ at which the speed of the bob is half of that at A, satisfies



(A) $\theta = \frac{\pi}{4}$ (B) $\frac{\pi}{4} < \theta < \frac{\pi}{2}$
(C) $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ (D) $\frac{3\pi}{4} < \theta < \pi$

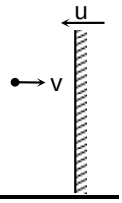
3. A particle B moving along x axis, starting from coordinate (x_0, 0) according to equation $x = (x_0 - a)e^{-bt} + ae^{ct}$, where x_0, a, b and c are positive constant and $2a > x_0 > a$ then choose correct statement.
(A) Velocity will go on decreasing with time
(B) Velocity will go on increasing with time
(C) Finally becomes zero velocity
(D) Direction of velocity will change at least one if c > b

4. A block of mass m is pushed towards a movable wedge of mass 2m and height h with a velocity u. All surfaces are smooth. The minimum value of u for which the block will reach the top of the wedge is



(A) $2\sqrt{gh}$ (B) $\sqrt{3gh}$
(C) $\sqrt{6gh}$ (D) $\sqrt{\frac{3}{2}gh}$

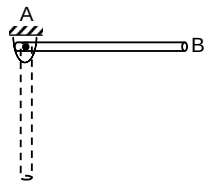
5. A ball moving with speed v normally towards a moving wall approaching it with speed u. The final speed of ball is (the collision is perfectly elastic)



(A) u+v
(B) 2u – v
(C) v + 2u
(D) will depend on the mass of the ball

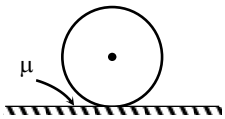
Space For Rough Work

6. One end of a uniform rod of length ℓ and mass m is hinged at A. It is released from rest from horizontal position AB as shown in figure. The force exerted by the rod on the hinge when it becomes vertical is



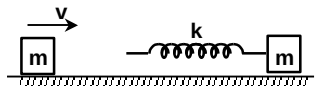
(A) 3/2 mg (B) 5/2 mg
(C) 3 mg (D) 5 mg

7. A disc of radius r is rotating about its centre with an angular speed ω_0. It is gently placed on a rough horizontal surface. After what time it will be in pure rolling?



(A) $\frac{\omega_0 r}{2\mu g}$ (B) $\frac{\omega_0 r}{3\mu g}$ (C) $\frac{\omega_0 r}{\mu g}$ (D) $\frac{3}{2}\frac{\omega_0 r}{\mu g}$

8. A mass m moving with velocity v collides a spring mass system at rest on a smooth floor as shown in the figure. The maximum force developed in the spring



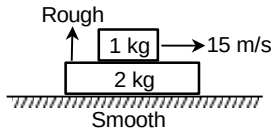
(A) $\sqrt{mk}\,v$ (B) $\sqrt{\frac{m}{k}}\,v$ (C) $\sqrt{\frac{m}{2k}}\,v$ (D) $\sqrt{\frac{mk}{2}}\,v$

Paragraph Type

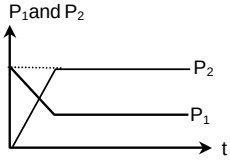
This section contains **4 paragraphs.** Based upon each paragraph, 2 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE is correct.**

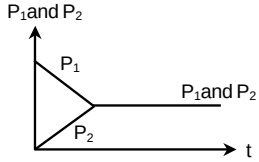
Paragraph for question 9 to 10

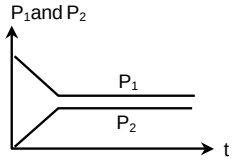
A 1 kg block is given a velocity of 15 m/s towards right over a very long rough plank of mass 2 kg placed on a horizontal smooth surface as shown in the figure.

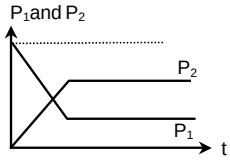


9. The correct graph showing linear momentum of 1 kg (i.e. P_1) and 2 kg (i.e. P_2) versus time is

(A)


(B)


(C)


(D)


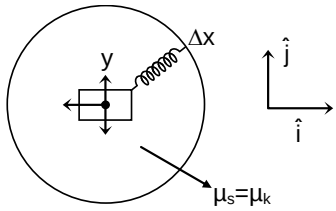
10. Momentum of both the blocks is equal at time

(A) 1.75 sec (B) 1.875 sec (C) 2.5 sec (D) 1.25 sec

Space For Rough Work

Paragraph for question 11 to 12

A block of mass 10 kg is placed at the centre of a rough disc which is at rest on a horizontal surface with its plane horizontal. The disc starts moving with a constant acceleration $\vec{a} = \frac{1}{2}\hat{i} + \frac{1}{2}\hat{j}\ m/s^2$ as shown in the figure. The coefficient of friction between the block and the disc is $\mu_S = \mu_K = 0.1$.



11. Now if an elongated spring of the spring constant k = 50 N/m is attached so that one end of spring is fixed to periphery of disc and other end to the block as shown in the figure, the bock just starts moving in +y (positive y axis) direction with negligible acceleration with respect to disc. The frictional force on block immediately after block starts moving with respect to disc is:

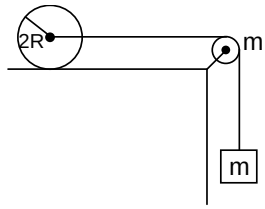
(A) $2N\left(-\hat{j}\right)$ (B) $5N\left(-\hat{j}\right)$ (C) $10N\left(-\hat{j}\right)$ (D) $15N\left(-\hat{j}\right)$

12. In the previous problem initial elongation in the spring is:

(A) $\frac{1}{\sqrt{10}}m$ (B) $\frac{1}{\sqrt{2}}m$ (C) $\frac{1}{\sqrt{5}}m$ (D) $\frac{3}{\sqrt{10}}m$

Paragraph for question 13 to 14

A uniform solid cylinder of mass m and radius 2R rests on a horizontal table. A string attached to it passes over a pulley (disc) of mass m and radius R that is mounted on a frictionless axle through its centre. A block of mass m is suspended from the free end of the string. The string does not slip over the pulley surface, and the cylinder rolls without slipping on the table.



13. Force of friction acting on the cylinder is.

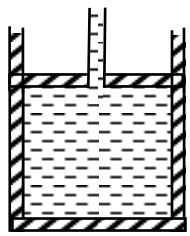
(A) $\frac{2mg}{3}$ (B) $\frac{3mg}{2}$ (C) $\frac{mg}{3}$ (D) $\frac{mg}{6}$

14. Angular acceleration of the cylinder is.

(A) $\frac{g}{3R}$ (B) $\frac{3g}{2R}$ (C) $\frac{g}{6R}$ (D) $\frac{g}{9R}$

Paragraph for question 15 to 16

A piston weighing 3 kg has the form of a circular disc of radius R = 4 cm. The disc has a hole into which a thin walled pipe of negligible mass of radius 1 cm is inserted. Initially the piston is at the bottom of the cylinder. Now 700 gm of water is poured into the pipe.



15. The height to which the piston will rise is:

(A) 5 cm (B) 10 cm
(C) 15 cm (D) 20cm

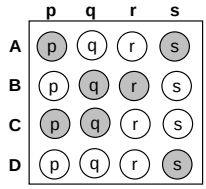
16. The length of liquid column in the pipe is approximately

(A) 50 cm (B) 54 cm (C) 60 cm (D) 64 cm

Space For Rough Work

PART-B : (Matrix-Match Type)

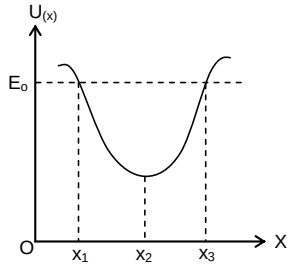
This Section contains **4 questions.** Each question has **four statements** (A, B, C and D) given in **Column-I** and **four statements** (p, q, r and s) in **Column II.** Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.



1. Net force on a system of particles in ground frame is zero. In each situation of Column-I a statement is given regarding this system. Match the statements in Column-I with results in Column-II.

	Column-I		Column-II
(A)	Acceleration of COM of system in ground frame.	(p)	Is constant.
(B)	Net momentum of system in ground frame.	(q)	Is zero.
(C)	Net momentum of system in the CM frame.	(r)	May be zero.
(D)	Kinetic energy of system in the CM frame.	(s)	May be constant.

2. The potential energy of a particle moving along x-axis is shown in figure. Where E_0 is the total mechanical energy of the particle.



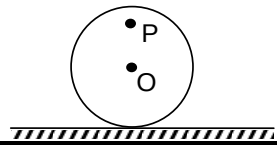
	Column-I		Column-II
(A)	Force on the particle is towards +ve x-axis if	(p)	$x_1 < x < x_2$
(B)	Particle can't be found where	(q)	$x_2 < x < x_3$
(C)	Force is towards the equilibrium position if	(r)	$x < x_1, x > x_3$
(D)	Force is towards the –ve x-axis if	(s)	$x_2 > x$

Space For Rough Work

3. Match the following and write the correct pairs.

	Column-I		Column-II
(A)	A block weighs 15 N in air and 12 N. When immersed in water. The specific gravity of the block is	(p)	$2^{5/3}$
(B)	Two equal drops of water are falling through air with a steady velocity 2 m/s. If the drops coalesce, the new velocity will be _____ m/s	(q)	1200
(C)	A solid shell loses half its weight in water. Relative density of shell is 5.0. What fraction of its volume is hollow?	(r)	3/5
(D)	A balloon has volume of 1000 m^3. It is filled with hydrogen (ρ = 0.09 g/l). If the density of air is 1.29 g/l, it can lift a total weight of _____ kg.	(s)	5

4. A uniform disc rolls without slipping on rough horizontal surface with uniform angular velocity. Point 'O' is centre of the disc & P is a point on the disc as shown in figure.



	Column-I		Column-II
(A)	The velocity of point P on the disc	(p)	Changes in magnitude with time.
(B)	The acceleration of point P on the disc	(q)	Always directed away from the point (the point on the disc given in Column I) towards the centre of the disc.
(C)	The tangential acceleration of the point P on the disc	(r)	Is always ZERO.
(D)	The acceleration of point on the disc which is in contact with rough horizontal surface.	(s)	Is non ZERO and remains constant in magnitude.

Space For Rough Work

SECTION –II: CHEMISTRY

PART–A (Single Correct Choice Type)

This section contains 8 multiple choice questions. Each question has 4 choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

1. A catalyst lowers the activation energy of a reaction from 20 $kJmole^{-1}$ to 10 $kJmole^{-1}$. The temperature at which the uncatalysed reaction will have the same rate as that of the catalysed at 27°C is

   (A) –123°C (B) 327°C (C) 600°C (D) + 23°C

2. $N_2 + 3H_2 \underset{}{\overset{\text{Catalyst}}{\rightleftharpoons}} 2NH_3$, $\Delta_r H° = -46.0$ KJ/mol, Kp=14

   Which is correct statement?

   (A) If N_2 is added at equilibrium condition? The equilibrium will shift to forward direction because according to 2^{nd} law of thermodynamics the entropy must increases in the direction of spontaneous reaction.

   (B) The condition for equilibrium is $G_{N_2} + 3G_{H_2} = 2G_{NH_3}$, where G is Gibbs free energy per mole of the gaseous species measured at that partial pressure. The condition of equilibrium is unaffected by the use of catalyst, which increases the rate of both the forward and backward reactions to the same extent.

   (C) The catalyst will increase the rate of forward reaction by α and that of backward reaction by β.

   (D) Catalyst will not alter the rate of either of the reaction.

3. Specific heat of a substance at melting point and specific heat of a substance during adiabatic change respectively are

   (A) 0,0 (B) 0, ∞ (C) ∞ ,0 (D) ∞, ∞

4. A sample of 100 ml of 0.10 M acid HA ($K_a = 1 \times 10^{-5}$) is titrated with standard 0.10 M KOH. How many ml of KOH is required to be added when the pH in the titration flask will be 5.00?

   (A) 0 ml (B) 10 ml (C) 100 ml (D) 50 ml

5. Which of the following statements about anhydrous aluminium chloride is correct?

   (A) It may fumes in moist air

   (B) It exists as dimer both in the vapour state below $350^0 C$ and in non polar solvents

   (C) It can be prepared by heating Al_2O_3 in a stream of S_2Cl_2 vapour and chlorine

   (D) All of these

space For Rough Work

6. PCl_5 decomposes as $PCl_5 \rightleftharpoons PCl_3 + Cl_2$. If at equilibrium, total pressure is P and density of the gaseous mixture be ρ at temperature T, M is molar mass of PCl_5, then degree of dissociation (α) would be

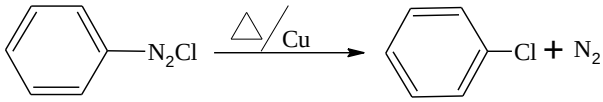
(A) $\alpha = 1 - \frac{PM}{\rho RT}$ (B) $\alpha = \frac{PM}{\rho RT} - 1$ (C) $\alpha = 1 - \frac{\rho RT}{PM}$ (D) $\alpha = \frac{\rho RT}{PM} - 1$

7. 1 mole of an ideal gas at 25^0C is subjected to expand reversibly ten times of its initial volume. The change in entropy during expansion is

(A) 19.15 $JK^{-1}mol^{-1}$ (B) 16.15 $JK^{-1}mol^{-1}$

(C) 22.15 $JK^{-1}mol^{-1}$ (D) none of these

8. Half life is independent of concentration of the reactant. After 10 minutes volumes of N_2 gas is 10L and after complete reaction it is 50L. Hence rate constant is



(A) $(2.303/10)\log 5\,min^{-1}$ (B) $(2.303/10)\log 1.25\,min^{-1}$

(C) $(2.303/10)\log 2\,min^{-1}$ (D) $(2.303/10)\log 4\,min^{-1}$

Comprehension Type

This section contains 4 paragraphs. Based upon each paragraph 2 or 3 multiple choice questions have to be answered. Each question has 4 choices (A), (B), (C) and (D), out of which ONLY ONE is correct.

Paragraph for Question 9 to 10

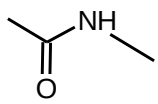
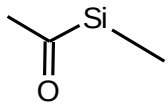
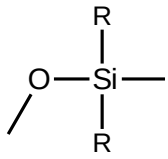
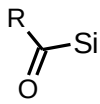
Read the paragraph carefully and answer the following questions:

Silicon is second most abundant element occurring in earth curst. It is always found in combined state. It is obtained by reduction of silica and exists in two allotropic forms, i.e. amorphous and crystalline. Amorphous form is obtained by heating dry powdered silica with magnesium. Crystalline form is obtained by heating sand or quartz in a furnace. Silicon is used in manufacturing of certain alloys and polymers such as ferrosilicon, silicon bronze etc.

The compound of silicon are carborundum, silicone, silicates, glass etc.
Silicones are organosilion polymer and silicate are metal derivatives of silicic acid. While carborundum are obtained by heating mixture of sand, carbon and common salt.

space For Rough Work

9. Which type of linkage is present in silicones?

(A)  (B)  (C)  (D) 

10. $(SiH_3)_3N$ is weaker base than $(CH_3)_3N$ because
(A) in$(SiH_3)_3N$, lone pair of electrons on nitrogen is involved in $d\pi-p\pi$ bonding
(B) in $(CH_3)_3N$ lone pair effect of three alkyl groups makes it least basic
(C) presence of $d\pi-p\pi$ back bonding in $(CH_3)_3N$
(D) In $(SiH_3)_3N$, high tendency to accept proton than $(CH_3)_3N$

Paragraph for Question 11 to 12

Read the paragraph carefully and answer the following questions:

A reaction is said to be of zero order if its rate is independent of the concentration of the reactants, i.e. the rate is proportional to the zeroth power of the concentration of the reactants. For a reaction,

$A \longrightarrow P$, to be of zero order,

$$\frac{dx}{dt} = K[A]^o$$

$$\therefore \frac{dx}{dt} = K$$

Some photochemical reactions and a few heterogeneous reactions are zero order reactions. Such reactions are not common.

11. For reaction, $2A \longrightarrow 3B$ with initial concentration of A as $2\,mole/\ell$ and rate constant $4 mole/\ell/hr$. The concentration of A after 30 min,
(A) Zero (B) $1.5\,mole/\ell$ (C) 1.25 M (D) 0.25 M

12. For any acid catalyzed reaction, $A \xrightarrow{H^+} B$. $t_{1/2}$ is independent of concentration of A at given pH. At same conc. of A, $t_{1/2}$ = 10 min at pH = 2 and $t_{1/2}$ = 100 min at pH=3. Find out the order of reaction.
(A) 1 (B) 2 (C) 3 (D) 0

space For Rough Work

Paragraph for Question 13 to 14

Read the paragraph carefully and answer the following questions:

A proper control of pH is very essential for many industrial as well as biological processes. Solution with a definite pH can be prepared from single salts or mixtures of acids/base and their salts. We also require solutions which resist change in pH and hence have a reserve pH value. Such solutions are called Buffer solutions.

For weak acid and base used by a chemist, data are given below:

$K_a = 1.8\times10^{-5}$, $K_b = 1.8\times10^{-5}$

Also logarithmic values of some numbers are given below:

log 1.8 = 0.2553, log 2 = 0.3010, log 3 = 0.4771, log 5 = 0.6990.

Report the approximate pH value in each of the following cases.

13. 100 mL of 0.10 M NaOH mixed with 100 ml of 0.05 M CH_3COOH solution

(A) 10.4 (B) 11.4 (C) 12.4 (D) 13.4

14. 100 mL of 0.10 M NH_4OH mixed with 100 mL of 0.05 M HCl solution

(A) 6.25 (B) 7.25 (C) 8.25 (D) 9.25

Paragraph for Question 15 to 16

Read the paragraph carefully and answer the following questions:

Rates of chemical reactions are strongly affected by temperature.

The rate constant for a reaction is:

$$\log K\ sec^{-1} = 14 - \frac{1.25\times10^4}{T}K$$

15. Pre-exponential factor for the above reaction is:

(A) 1.25×10^{-14} sec^{-1} (B) 10^{-14} sec^{-1} (C) 10^{14} sec^{-1} (D) 14 sec^{-1}

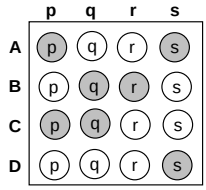
16. Energy of activation of this reaction will be

(A) 239.4 Kcal mol^{-1} (B) 57.2 Kcal mol^{-1}

(C) 1.25×10^4 KJ mol^{-1} (D) 239.4 J mol^{-1}

Part–B (Matrix– Match Type)

This Section contains **4 questions.** Each question has **four statements** (A, B, C and D) given in **Column-I** and **four statements** (p, q, r and s) in **Column II.** Any given statement in Column I can have correct matching with **ONE** or **MORE** statement(s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.



space For Rough Work

1. Match the following

	Column – I (solution mixed)		Column – II (Normality after mixing) for H^+
A)	100 cc of 0.2N H_2SO_4 + 100 ml of 0.1 N HCl	p)	0.25 N
B)	100 cc of 0.2 M H_2SO_4+100 cc of 0.1 M HCl	q)	0.014 N
C)	100 cc of 0.1 M H_2SO_4 + 100 cc of 0.1M NaOH	r)	0.15 N
D)	100 cc of 0.1 M HCl + 40 cc of 0.2 N NaOH	s)	0.05 N

2. Match the following

	Column – I		Column – II
A)	Borax $\xrightarrow{\Delta}$	p)	BN
B)	$B_2H_6 + H_2O \xrightarrow{\Delta}$	q)	B_2H_6
C)	$B_2H_6 + NH_3(\text{excess}) \xrightarrow{\Delta}$	r)	H_3BO_3
D)	$BCl_3 + LiAlH_4 \longrightarrow$	s)	$NaBO_2$+B_2O_3

3. Match the following

	Column – I		Column – II
A)	$NaHCO_3(aq)$	P	Cationic hydrolysis
B)	$NH_4CN(aq)\left(K_a(HCN)=10^{-10}, K_b(NH_3)=10^{-5}\right)$	Q	Anionic hydrolysis
C)	$NH_4Cl(aq)$	R	Acidic solution
D)	$NaCN(aq)$	S	Basic solution

4. Match the following

	Column – I (Piston)		Column – II
A)	Isobaric process	p)	$\Delta Q > 0$
B)	Isothermal process	q)	$\Delta Q < 0$
C)	Isochoric process	r)	W > 0
D)	Adiabatic process	s)	W < 0

space For Rough Work

SECTION – III : MATHEMATICS

PART– A (Single Correct Choice Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

1. If α and β are the roots of $a(x^2-1)+2bx=0$ then, which one of the following are the roots of the same equation?

(A) $\alpha+\beta,\alpha-\beta$ (B) $2\alpha+\frac{1}{\beta},2\beta+\frac{1}{\alpha}$ (C) $\alpha+\frac{1}{\beta},\beta-\frac{1}{\alpha}$ (D) $\alpha+\frac{1}{2\beta},\beta-\frac{1}{2\alpha}$

2. If α, β are the roots of the quadratic equation $x^2-(3+2^{\sqrt{\log_2 3}}-3^{\sqrt{\log_3 2}})x-2(3^{\log_3 2}-2^{\log_2 3})=0$ then the value of $\alpha^2+\alpha\beta+\beta^2$ is equal to:

(A) 3 (B) 5 (C) 7 (D) 11

3. Let $s_1+s_2+s_3$ ….. and $t_1+t_2+t_3$ …. are two arithmetic sequences such that $s_1=t_1\neq 0$; $s_2=2t_2$ and $\sum_{i=1}^{10}s_i=\sum_{i=1}^{15}t_i$. Then the value of $\frac{s_2-s_1}{t_2-t_1}$ is:

(A) $\frac{8}{3}$ (B) $\frac{3}{2}$ (C) $\frac{19}{8}$ (D) 2

4. Three equal circles each of radius r touch one another. The radius of the circle which is touched by all the three given circles is internally is

(A) $(2+\sqrt{3})r$ (B) $\frac{(2+\sqrt{3})}{\sqrt{3}}r$ (C) $\frac{(2-\sqrt{3})}{\sqrt{3}}r$ (D) $(2-\sqrt{3})r$

5. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1<0$, $y_2<0$, be the end points of the latus rectum of the ellipse $x^2+4y^2=4$. The equations of parabolas with latus recta PQ are

(A) $x^2+2\sqrt{3}y=3+\sqrt{3}$ (B) $x^2-2\sqrt{3}y=3+\sqrt{3}$

(C) $x^2+2\sqrt{3}y=3-\sqrt{3}$ (D) $x^2-2\sqrt{3}y=3-\sqrt{3}$

Space For Rough Work

6. If B, C are images of point A(1,4) with respect to y = 2x+1 and 2y=x+1 respectively, then circumcentre of ΔABC is

(A) $\left(-\frac{2}{3},\frac{1}{3}\right)$ (B) $\left(\frac{1}{3},-\frac{1}{3}\right)$ (C) $\left(-\frac{1}{3},\frac{1}{3}\right)$ (D) $\left(\frac{2}{3},\frac{1}{3}\right)$

7. If $A\left(\alpha,\frac{1}{\alpha}\right),B\left(\beta,\frac{1}{\beta}\right),C\left(\gamma,\frac{1}{\gamma}\right)$ be he vertices of a ΔABC where α,β be the roots of $p_3^2-6p_3+6=0$; β, γ be the roots of $p_2^2-6p_2+3=0$; γ, α be the roots of $p_3^2-6p_3+6=0$ Where p_1, p_2, $p_3 \in I^+$, then centroid of ΔABC is:

(A) $\left(2,\frac{1}{18}\right)$ (B) $\left(\frac{11}{18},2\right)$ (C) $\left(2,\frac{11}{18}\right)$ (D) $\left(\frac{1}{18},2\right)$

8. If α and β are the roots of the equation $(\log_2 x)^2 + 4(\log_2 x)-1=0$ then the value of $\log_\beta \alpha+\log_\alpha \beta$ equals:

(A) 18 (B) -16 (C) 14 (D) -18

Section–2 : Comprehension Type (Only one option correct)

This section contains 4 paragraphs, each describing theory, experiments, data etc. Six questions relate to the three paragraphs with two questions on each paragraph. Each question has only one correct answer among the four given option (A), (B), (C) and (D).

Paragraph for Question Nos. 9 to 10

Let $A_1, A_2, A_3, \ldots, A_r$ be arithmetic means between –2 and 1027 and $G_1, G_2, G_3, \ldots G_k$ be geometric means between 1 and 1024. Product of geometric means is 2^{45} and sum of arithmetic means is 1025×171.

9. The value of k is

(A) 13 (B) 11 (C) 9 (D) 7

10. The common difference of the progression $A_1, A_3, A_5, \ldots, A_{r-1}$ is

(A)1 (B) 2 (C) 3 (D) 6

Space For Rough Work

Paragraph for Question Nos. 11 to 12

Let two parabolas P_1 and P_2 are $P_1 : y^2 - 4ax = 0$ and $P_2 : x^2 - 4by = 0$ and a straight line is $L : y = mx + c$, then answer the following problems

11. If line L is a tangent to P_1 and P_2 both, then:

(A) $m = \frac{b}{a}$ (B) $m = -\frac{b}{a}$ (C) $m = \left(\frac{b}{a}\right)^{1/3}$ (D) $m = -\left(\frac{a}{b}\right)^{1/3}$

12. Locus of mid-point of any focal chord of parabola P_2 is:

(A) $x^2 = 5b(y-b)$ (B) $x^2=2b(y-b)$ (C) $y^2 = 2b(x-b)$ (D) $y^2 = 4b(x-b)$

Paragraph for Question Nos. 13 to 14

The graph of the conic $x^2 - (y-1)^2 = 1$ has one tangent line with positive slope that passes through the origin. The point of tangency being (a, b). Then:

13. The value of $\sin^{-1}\left(\frac{a}{b}\right)$ is:

(A) $\frac{5\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

14. Length of the latus rectum of the conic is :

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) none of these

Paragraph for Question 15 and 16

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B.

15. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is

(A) $2x - \sqrt{5}y - 20 = 0$ (B) $2x - \sqrt{5}y + 4 = 0$

(C) $3x - 4y + 8 = 0$ (D) $4x - 3y + 4 = 0$

16. Equation of the circle with AB as its diameter is

(A) $x^2 + y^2 - 12x + 24 = 0$ (B) $x^2 + y^2 + 12x + 24 = 0$

(C) $x^2 + y^2 + 24x - 12 = 0$ (D) $x^2 + y^2 - 24x - 12 = 0$

Space For Rough Work

PART-B : (Matrix-Match Type)

This section contains **4 questions**. Each question has **four statements** (A, B, C and D) given in **Column I** and **four statements** (p, q, r and s) in **Column II.** Any given statement in **Column I** can have correct matching with one or more statement(s) in **Column II.** For example, if for a given question, statement B matches with the statements given in q and r, then for that particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

1. Match the following and write the correct pairs.

	Column I		Column II
(A)	Largest integral value of 'x' satisfying $\frac{(x+2)^2(2x-1)^3}{(x^2+1)(x-3)} < 0$ is	(p)	0
(B)	Greatest value of the expression $\frac{x^2+1}{x^2+x+1}, x \in R$ is	(q)	4
(C)	Number of integral value(s) of 'a' for which 2 lies between the roots of equation $(2a-1)x^2 + x-1 = 0$, is	(r)	3
(D)	If sin x, cos y and sin z are in G.P. and $x+z=\frac{\pi}{2}$, then the value of $\frac{\cos(x-z)}{\cos^2 y}$, whenever exists is	(s)	2

2. Match the following and write the correct pairs.

	Column I		Column II
(A)	If the mid point of a chord of the ellipse $\frac{x^2}{16}+\frac{y^2}{25}=1$ is (0,3), then length of the chord is $\frac{4k}{5}$, then k is	(p)	6
(B)	If the line $y = x + \lambda$ touches the ellipse $9x^2+16y^2=144$, then the sum of values of λ is	(q)	8
(C)	If the distance between a focus and corresponding directrix of an ellipse be 8 and the eccentricity be $\frac{1}{2}$, then length of the minor axis is $\frac{k}{\sqrt{3}}$, then k is	(r)	0
(D)	Sum of distances of a point on the ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ from the foci is	(s)	16

Space For Rough Work

3. Match the following and write the correct pairs.

	Column I		Column II
(A)	Normals are drawn to the parabola $y^2 = 4x$ at $P(t^2, 2t)$; where $t < 0$, which meets a the curve again at Q. If ordinate of Q is minimum, then PQ^2 is	(p)	3
(B)	From point on circle $x^2+y^2 = 36$, tangents are drawn to hyperbola $x^2-y^2=36$, locus of mid-points of chords of contact is $(x^2-y^2)^2 = 12\ell(x^2+y^2)$, then ℓ =	(q)	0
(C)	Maximum length of chord of ellipse $\frac{x^2}{2}+y^2=1$, such that eccentric angles of its extremities differ by $\frac{\pi}{2}$ is	(r)	108
(D)	Three distinct lines are drawn in a plane and there are exactly n circles in plane tangent to all three lines, then n can be	(s)	2

4. Match the following and write the correct pairs.

	Column I		Column II
(A)	The minimum value of the sum of real numbers $a^{-5}, a^{-4}, 3a^{-3}, 1, a^8$ and a^{10} with $a > 0$ is	(p)	4
(B)	Let a, b, c be positive integers such that $\frac{a}{b}$ is an integer. If a, b, c are in geometric progression and the arithmetic mean of a, b, c is b + 2, then the value of $\frac{a^2+a-14}{a+1}$ is	(q)	3
(C)	Let α and β be the roots of $x^2-6x-2=0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{2a_9}$ is	(r)	8
(D)	Let (x_0, y_0) be the solution of the following equations $(2x)^{\ln 2} = (3y)^{\ln 3}, 3^{\ln x} = 2^{\ln y}$. Then $2x_0$ is	(s)	1

Space For Rough Work

FIITJEE COMMON TEST

BATCHES: Two Year CRP (1820) B LOT

PHASE TEST-II: PAPER-II

JEE ADVANCED LEVEL

ANSWER KEY

Paper Code
SET-A

SECTION – I (PHYSICS)

PART – A

1. D P110414
2. D P110503
3. B P110305
4. B P110604
5. C P110606
6. B P111816
7. B P111822
8. D P110608
9. D P110604
10. B P110604
11. C P110411
12. A P110411
13. D P111826
14. C P111826
15. B P111001
16. D P111001

PART – B

1. A–p,q; B–p,r; C–p,q; D–r,s P110612
2. A–p,s; B–r; C–p,q, s; D–q P110501
3. A–s; B–p; C–r; D–q P111005
4. A–p; B–q; C–p; D–q, s P111826

SECTION – II (CHEMISTRY)

PART–A

1. B C110603
2. B C110403
3. C C123204
4. D C110504
5. D C111510
6. B C110402
7. A C123204
8. B C110601
9. C C111607
10. A C111607
11. A C122111
12. B C123403
13. C C110502
14. D C110504
15. C C123404
16. B C123404

Part–B

1. A–r;B–p,C–s,D–q, C110505
2. A–s, B–r, C–p, D–q C111506
3. A–q,s; B–p,q,s;C–p,r; D–q,s C110503
4. A–p,q,r,s; B–p,q,r,s; C–p,q ; D–r,s C123201

SECTION – III (MATHS)

PART–A

1. B M110101
2. C M110101
3. C M110501
4. B M111511
5. C M111001
6. C M110738
7. C M110103
8. D M110202
9. C M110504
10. D M110501
11. D M110903
12. B M110921
13. D M111106
14. C M111103
15. B M111106
16. A M110802

Part–B

1. A-s, B-s,C-p, D-s M112402, M110115, M110109, M111402
2. A-q, B-r, C-s, D-q M111001
3. A-r, B-p, C-s, D-q,s M110918, M110809, M111014, M110707
4. A-r, B-p, C-q, D-s M110502, M110503, M110101, M110201

FIITJEE COMMON TEST

BATCHES: Two Year CRP (1820) B LOT (CPT2-1)

PHASE TEST-II: PAPER-I

HINT & SOLUTIONS : PHYSICS

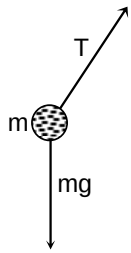
PART – A (Single Correct Choice Type)

1. D P110414

$T\cos\theta = mg$

$T\sin\theta = m\omega^2 L\sin\theta$

$\omega = \sqrt{T/mL} = \sqrt{\dfrac{324}{0.5\times 0.5}} = 36$



2. D P110503

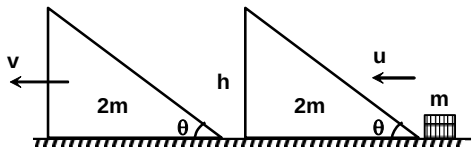
$v_A = \sqrt{5gL}$ energy conservation

3. B P110305

$v = \dfrac{dx}{dt} = -b(x_0 - a)e^{-bt} + ace^{ct}$

4. B P110604

Block just reaches the top of the wedge, it implies that velocity of block with respect to wedge at the top of the wedge is zero. Let v be the horizontal velocity of both at this instant. Then from conservation of linear momentum, we have (2m + m)v = mu



Now from conservation of mechanical energy, we get $\dfrac{1}{2}mu^2 = \dfrac{1}{2}(3m)v^2 + mgh$

5. C P110606

e = (separation speed)/(approach speed) for e = 1

v' – v = v + u $\Rightarrow$ v' = 2u + v

6. B P111816

R – mg = $m\omega^2$ (ℓ/2) and $\dfrac{1}{2}\dfrac{m\ell^2}{3}\omega^2 = mg\dfrac{\ell}{2}$

7. B P111822

$\dfrac{1}{2}mr^2\omega_0 = mvr + \dfrac{1}{2}mr^2\dfrac{v}{r} \Rightarrow v = \dfrac{r\omega_0}{3}; \dfrac{r\omega_0}{3} = 0 + \mu gt \Rightarrow t = \dfrac{\omega_0 r}{3\mu g}$

8. D P110608

mv = (m +m) v' $\Rightarrow$ v' = v/2

$$\frac{1}{2}mv^2 = \frac{1}{2}(m+m)v' + \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{mv^2}{2k}} = \sqrt{\frac{mk}{2}}\,v$$

9. D P110604

10. B P110604

Use conservation of linear momentum

11. C P110411

12. A P110411

Sol: 11–12.

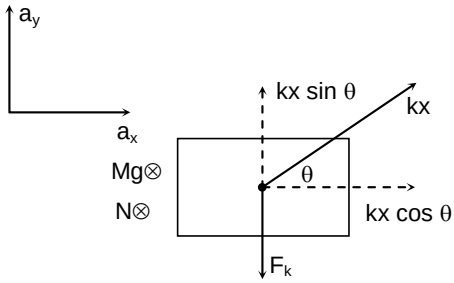
Since three is a relative motion between block and disc, so the nature of friction force will be kinetic and it will act along negative y-axis.

FBD of block is ground from

$kx\cos\theta = m \times ½ = 5$

$kx\sin\theta - 10 = m \times ½$

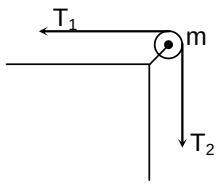
$\Rightarrow \quad kx\sin\theta = 15$



13. D P111826

14. C P111826

Sol.13-14



15. B P111001

Volume in pipe = $64\pi 1^2$ = 200 cc

Volume in cylinder = 500 cc = $h\pi 4^2$

16. D P111001

$$\rho g h_1 = \frac{3g}{\pi\left(4^2 - 1^2\right)}$$

PART - B : (Matrix-Match Type)

1. A–p,q; B–p,r; C–p,q; D–r,s P110612

$\vec{F}_{ext} = 0 \Rightarrow \vec{a}_{CM} = 0 \Rightarrow a_1$

$\therefore \vec{P}_{CM} = \text{constant.} \Rightarrow \vec{P}_1 + \vec{P}_2 + ... = \text{constant}$

In C.M from

$\vec{P}_{CM} = 0 \Rightarrow \vec{P}_1 + \vec{P}_2 + ... = 0$

K.E. of system depends upon internal forces also.

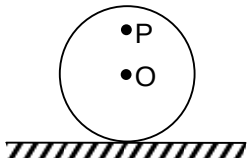
2. A–p,s; B–r; C–p,q, s; D–q P110501

Force F = (–) $\frac{dU}{dx}$

3. A–s; B–p; C–r; D–q P111005

$$\vec{B} = -V_s\rho_f\vec{g};\ v_t = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

4. A–p; B–q; C–p; D–q, s P111826



The velocity of point P on the does C change in magnitude with time.

Since $\vec{V}_p = \vec{V}_o + \vec{V}_{po}$

V_o is in straight line

In position I, $\vec{V}_{p1} = \vec{V}_o + \vec{V}_{po} \Rightarrow \left|\vec{V}_{p1}\right| = \sqrt{V_o^2 + (\omega r)^2}$

SECTION-II (CHEMISTRY) HINT AND SOLUTION

PART–A

1. B C110603

$\frac{E_a'}{T_1} = \frac{E_a}{T_2} = \frac{10}{300} = \frac{20}{T_2}$

$\therefore T_2 = 600\ K = 327°C$

2. B C110403

At equilibrium $\Delta_r G = 0$ and no effect of catalyst at equilibrium

3. C C123204

$q = m.s.\Delta T$ if $\Delta T = 0$ (at melting point), $S=\infty$, if $q= 0$ (adiabatic condition), $S=0$

4. D C110504

$pH = pK_a + \log\frac{[salt]}{[acid]}$

$5 = 5 + \log\left(\frac{N_2V_2}{N_1V_1 - N_2V_2}\right)$

$1 = \frac{0.1\times V_2}{0.1\times 100 - 0.1\times V_2}$

$10 - 0.1V_2 = 0.1\ V_2$

$V_2 = 50$ ml

5. D C111510

All are correct statements

6. B C110402

$\alpha = \frac{D-d}{(n-1)d}$, D = vapour density when there is no dissociation, d= vapour density when degree of dissociation is α., $\therefore P\times 2d=\rho RT$ apply

7. A C123204

$\Delta S = 2.303\ nR \log V_2/V_1$

8. B C110601

$Kt=2.303 \log V_\infty\backslash[V_\infty - Vt]$

9. C C111607

10. A C111607

11. A C122111

$K=\frac{A_0 - A}{t}$, it is zero order reaction

12. B C123403

13. C C110502

CH_3COOH mm = 5 and NaOH mm = 10

$\therefore$ resultant solution is NaOH(mm = 5) + CH_3COOH (mm=5), $\therefore$ M= 0.025(NaOH)

pOH= 1.6 $\therefore$pH=12.4

14. D C110504

Resultant solution will be NH_4OH (mm=5) + NH_4Cl(mm=5) it is basic buffer solution.

$pOH = pK_b + \log\frac{[NH_4^+]}{[NH_4OH]}$

15. C C123404

$\therefore \log k = \log A - \frac{Ea}{2.303RT}$ $\therefore \log A = 14$

16. B C123404

Part–B

1. A–r; B–p, C–s, D–q, C110505

a) 100 cc of 02 N H_2SO_4 = 100×0.2 meq = 20 meq
100 cc of 0.1 N HCl = 100×0.1 meq = 10 meq
Total meq = 30 ; Total volume = 200 cc

$\therefore$ Normality = $\frac{\mathbf{30\ meq}}{\mathbf{200\,cc}} = \mathbf{0.15N}$

b) 100 cc of 0.2 M H_2SO_4 = 100×0.2 mol
= 20 m mol = 40 meq
100 cc of 0.1 M HCl = 100×0.1 m mol = 10 m.eq

Normality = $\frac{\mathbf{50\ meq}}{\mathbf{200\ cc}} = \mathbf{0.25N}$

c) 100 cc of 0.1 M H_2SO_4 = 20 meq
100 cc of 0.1 M NaOH = 10 meq.
10 meq NaOH will neutralize 10 meq of H_2SO_4
H_2SO_4 left = 10 meq, total volume = 200 cc

$\therefore$ Normality = $\frac{\mathbf{10\ meq}}{\mathbf{200\ cc}} = \mathbf{0.05\ N}$

d) 100 cc of 0.1 N HCl = 10 meq
40 cc of 0.2 N NaOH = 8 meq

Normality of HCl = $\frac{\mathbf{2\ meq}}{\mathbf{140\,cc}}$ =0.014 N

2. A–s, B–r, C–p, D–q C111506
3. A–q,s; B–p,q,s; C–p,r; D–q,s C110503
4. A–p,q,r,s; B–p,q,r,s; C–p,q ; D–r,s C123201

MATHEMATICS
HINT & SOLUTIONS

1. B M110101
Verify and eliminate each option by checking with product of roots.
2. C M110101

We know that $2^{\sqrt{\log_2 3}} = 2^{(\log_2 3)\left(\frac{1}{\sqrt{\log_2 3}}\right)}$

$= (2^{\log_2 3})^{\frac{1}{\sqrt{\log_2 3}}} = 3^{\sqrt{\log_3 2}}$

(Using base changing formula)
$\therefore$ The given equation becomes $x^2 - 3x + 2 = 0$
$\Rightarrow \alpha=1, \beta = 2$
Hence $\alpha^2 + \alpha\beta + \beta^2 = 7$

3. C M110501
Given $s_1, s_2, s_3 + \ldots + s_{10} = t_1, t_2, t_3 + \ldots + t_{15}$ let 1st sequence is $a_1, a_1 + d_1, a_1 + 2d_1, \ldots$ and 2nd is $a_1, a_1 + d_2, a_1 + 2d_2, \ldots$ (since $s_1 = t_1$)
given $s_2 = 2t_2$
$\therefore\ a_1 + d_1 = 2(a_1 + d_2)$
$\therefore\ a_1 = d_1 - 2d_2$ ….(i)

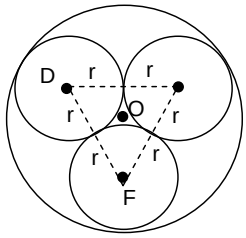
We have to find $\frac{s_2 - s_1}{t_2 - t_1} = \frac{d_1}{d_2} = ?$

Now $\frac{10}{2}[2a_1 + 9d_1] = \frac{15}{2}[2a_1 + 14d_2]$

This gives $a_1 = 9d_1 - 21d_2$ ….(ii)

From equations (i) and (ii), $\frac{d_1}{d_2} = \frac{19}{8}$.

4. B M111511



$\because$ ΔDEF is equilateral with side 2r if radius of circumcircle DEF is R_1, then are of $\Delta DEF = \frac{\sqrt{3}}{4}(2r)^2 = \sqrt{3}r^2$

$$\sqrt{3}r^2 = \frac{2r.2r.2r}{4R_1}$$

$$\Rightarrow R_1 = \frac{2r}{\sqrt{3}}$$

$\therefore$ Radius of the circle touching all the three given circles $= r + R_1$

$$= r + \frac{2r}{\sqrt{3}} = \frac{(2+\sqrt{3})r}{\sqrt{3}}$$

5. C M111001

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

$a^2=4$, $b^2=1$

$1=4(1-e^2)$

$e^2 = ¾$

$$e = \frac{\sqrt{3}}{2}$$

$$P\left(-ae, \frac{-b^2}{a}\right), Q\left(ae, \frac{-b^2}{a}\right) \quad \Rightarrow P\left(-\sqrt{3}, \frac{-1}{2}\right), Q\left(\sqrt{3}, -\frac{1}{2}\right)$$

Focus of required parabola = (0, –1/2)

Length of latus rectum of parabola = $2\sqrt{3}$

$\Rightarrow$ distance between focus and directrix = $\sqrt{3}$.

Also slope of directrix = slope of latus rectum = 0

So equation of directrix is either.

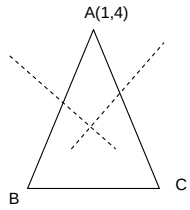
$$y + \frac{1}{2} - \sqrt{3} = 0 \quad \text{or} \quad y + \frac{1}{2} + \sqrt{3} = 0$$

$\Rightarrow$ equation of parabola is

either $\sqrt{x^2 + \left(y + \frac{1}{2}\right)^2} = \left|y + \frac{1}{2} - \sqrt{3}\right|$ or $\sqrt{x^2 + \left(y + \frac{1}{2}\right)^2} = \left|y + \frac{1}{2} + \sqrt{3}\right|$

6. C M110738

Circumcentre is point of intersection of perpendicular bisectors of sides



So, solving y = 2x + 1 and 2y = x + 1, we get $x=-\frac{1}{3}, y=\frac{1}{3}$

7. C M110103

$\alpha\beta=2, \beta\gamma=3, \gamma\alpha=6$

$\alpha^2\beta^2\gamma^2=36$

$\Rightarrow \alpha\beta\gamma = 6$

$\alpha = 2$, $\beta = 1$ and $\gamma = 3$

$\therefore$ $A\left(2,\frac{1}{2}\right), B(1,1)$ and $C\left(3,\frac{1}{3}\right)$

Hence, centroid of ΔABC is $\left(2,\frac{11}{18}\right)$

8. D M110202

$\log_2\alpha+\log_2\beta=-4; \log_2\alpha\cdot\log_2\beta=-1$

Now $\log_\beta\alpha+\log_\alpha\beta=\frac{\log_2\alpha}{\log_2\beta}+\frac{\log_2\beta}{\log_2\alpha}$

$= \frac{(\log_2\alpha)^2+(\log_2\beta)^2}{\log_2\alpha\cdot\log_2\beta}$

$=-[(\log_2\alpha+\log_2\beta)^2-2\log_2\alpha.\log_2\beta]$

= –[16 + 2] =–18

9. C M110504

$G_1\times G_2\times\ldots.\times G_k=(ab)^{k/2}=\left(\sqrt{1\times1024}\right)^k=2^{5k}$

$2^{5k} = 2^{45}$

$\Rightarrow k = 9$

10. D M110501

$\frac{r}{2}\times1025=1025\times171\Rightarrow r=342$

$A_1, A_3, A_5, \ldots, A_{341}$

Common difference = 2d = ?

$1027 = T_{344}$

$1027 = -2 + (343).d$

$\frac{1029}{343}=d\Rightarrow d=3$

$\Rightarrow 2d = 6$

11. D M110903

$y = mx + a/m$ [tangent of P_1]

$y = mx - bm^2$ [tangent of P_2]

$\Rightarrow a/m = -bm^2 \rightarrow m = -(a/b)^{1/3}$

12. B M110921

Let mid-point be (h.k)

$\Rightarrow h = \dfrac{2bt_1 + 2bt_2}{2}$ .....(i)

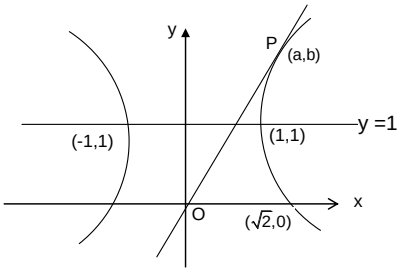
$k = \dfrac{bt_1^2 + bt_2^2}{2}$ .....(ii)

and $t_1t_2 = -1$ .....(iii)

By using three equations, eliminate t_1 and t_2.

13. D M111106

Differentiate the curve



$2x - 2(y-1)\dfrac{dy}{dx} = 0$

$\left.\dfrac{dy}{dx}\right]_{(a,b)} = \dfrac{a}{b-1} = \dfrac{b}{a}\left(m_{OP} = \dfrac{b}{a}\right)$

$a^2 = b^2 - b$ ....(i)

Also (a, b) satisfy the curves,

$a^2 - (b-1)^2 = 1$

$a^2 - (b^2 - 2b + 1) = 1$

$a^2 - b^2 + 2b = 2$

$-b + 2b = 2$

$\Rightarrow b = 2$ { putting $a^2 - b^2 = -b$ from eqn.(i) }

$\therefore a = \sqrt{2}$

$\therefore \sin^{-1}\left(\dfrac{a}{b}\right) = \dfrac{\pi}{4}$

14. C M111103

Length of latus rectum $\dfrac{2b^2}{a} = 2a =$ distance between the vertices = 2

(note that the hyperbola is rectangular)

15. B M111106

Equation of tangent of hyperbola is $y = mx \pm \sqrt{9m^2 - 4}$

it is also the tangent of circle, hence

$\left|\dfrac{4m \pm \sqrt{9m^2 - 4}}{\sqrt{1+m^2}}\right| = 4 \Rightarrow m = \dfrac{2}{\sqrt{5}}$

hence tangent is $2x - \sqrt{5}y + 4 = 0$

16. A M110802

Intersection point of $x^2 + y^2 - 8x = 0$ & $4x^2 - 9y^2 = 36$

$4x^2 - 9(8x - x^2) = 36$

$\Rightarrow 13x^2 - 72x - 36 = 0$

$\Rightarrow x = 6,\ x = \dfrac{-6}{13}$ rejecting

$x = 6,\ y^2 = 12 \Rightarrow y = \pm 2\sqrt{3}$

$A(6,\ 2\sqrt{3}),\ B(6,\ -2\sqrt{3})$

$\therefore$ Equation of circle with AB as diameter

$(x-6)^2 + (y^2 - 12) = 0 \Rightarrow x^2 + y^2 - 12x + 24 = 0$

1. A-s, B-s, C-p, D-s M112402, M110115, M110109, M111402

A. $x \in \left(\frac{1}{2}, 3\right)$ and $x_{max} = 2$

B. $y = \frac{x^2+1}{x^2+x+1} \Rightarrow (1-y)x^2 - yx + (1-y) = 0$

$D \geq 0 \Rightarrow \frac{2}{3} \leq y \leq 2$

C. $(2a-1).(8a-1)<0$

$\Rightarrow \frac{1}{8} < a < \frac{1}{2}$

D. $\cos^2 y = \sin x.\sin z$

$\Rightarrow \cos^2 y = \frac{1}{2}(\cos(x-z) - \cos(x+z))$

$\Rightarrow \frac{\cos(x-z)}{\cos^2 y} = 2$

2. A-q, B-r, C-s, D-q M111001

A. Equation of the chord whose mid point is (0,3) is

$\frac{3y}{25} - 1 = \frac{9}{25} - 1$ i.e. $y = 3$

If intersects the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$

at $\frac{x^2}{16} = 1 - \frac{9}{25} = \frac{16}{25} \Rightarrow x = \pm\frac{16}{5}$ thus $\frac{4k}{5} = \frac{32}{5}$ $\therefore k = 8$

B. If the line $y = x + \lambda$ touches the ellipse $9x^2+16y^2=144$

Then $\lambda^2 = 16(1)^2 + 9 \Rightarrow \lambda = \pm 5$

C. $\frac{a}{e} - ae = 8 \Rightarrow 2a - \frac{a}{2} = 8$ i.e. $a = \frac{16}{3}$

$b^2 = a^2(1-e^2) = \frac{256}{a}\left(1 - \frac{1}{4}\right) = \frac{64}{3}$ $\therefore$ length of minor axis $= 2b = \frac{16}{\sqrt{3}} = \frac{k}{\sqrt{3}}$

$\therefore k = 16$

D. Sum of the distance of a point on the ellipse from the foci = 2a = 2×4 = 8

3. A-r B-p C-s D-q,s M110918, M110809, M111014, M110707

A $\because$ Point P is $(t^2, 2t)$ and Q is $\left[\left(-t - \frac{2}{t}\right)^2, 2(-t - 2/t)\right]$

$-t - \frac{2}{t} \geq 2\sqrt{2}$

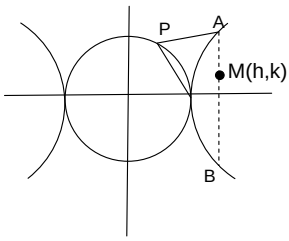
For minimum, $t = -\sqrt{2}$

$\therefore$ Points are $P(2, -2\sqrt{2})$ and $Q(8, 4\sqrt{2})$

$\therefore PQ^2 = 108$

B $\because$ Equation of chord of contact is $T = 0$

$x\cos\theta - y\sin\theta = 6$



$x\cos\theta - y\sin\theta = 6$ ….(i)

Let M(h, k) is mid-point of AB

$\therefore$ Let M(h,k) is mid-point of AB

$\therefore$ Equation of AB is $T = S_1$

$\Rightarrow hx - ky = h^2 - k^2$ …..(ii)

$\therefore$ Comparing equations (i) and (ii)

$$\cos\theta = \frac{6h}{h^2 - k^2}$$

$$\sin\theta = \frac{6k}{h^2 - k^2}$$

Square and add, locus is $(x^2 - y^2)^2 = 36(x^2+y^2)$

$\therefore \ \ell = 3$

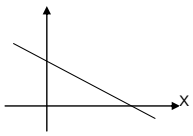
C. Let $P(\sqrt{2}\cos\theta, \sin\theta)$ and $Q\left(-\sqrt{2}\sin\theta, \cos\theta\right)$ are extremities of chord

$\therefore$ Length $PQ = \sqrt{3 + \sin 2\theta} \le 2$

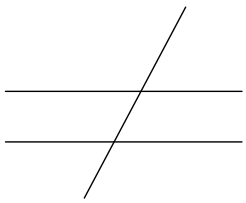
$\therefore (PQ)_{max} = 2$

D. Case(i) : If lines form a triangle then n = 4

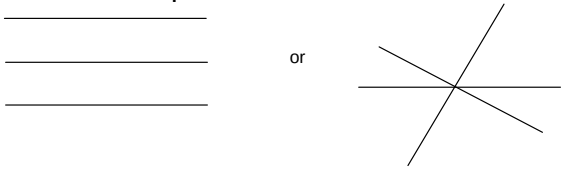
i.e. 3 excircle and 1 incircle



(ii) Two lines are parallel and third intersect them then n = 2



(iii) All lines are parallel or lines are concurrent there n = 0



4. A-r, B-p, C-q, D-s M110502, M110503, M110101, M110201

A 8

Use AM $\ge$ GM

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8} \ge \left(a^{-5}.a^{-4}.a^{-3}.a^{-3}.a^{-3}.1.a^8.a^{10}\right)^{\frac{1}{8}}$$

$$\Rightarrow a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10} \ge 8$$

Hence minimum value = 8

B 4

$\frac{a+b+c}{3} = b+2$

$\Rightarrow a + c = 2b + 6$

a, b, c are in G.P.

Let a = x be +ve integer

b = rx

$c = r^2x$

$\Rightarrow r^2x - 2rx + x - 6 = 0$

$x(r-1)^2 = 6$

x, r are integers

$\therefore$ r = 2

$\Rightarrow x = 6 = a$

So $\frac{a^2+a-14}{a+1} = \frac{36+6-14}{6+1} = 4$

C C

$a_{n+1} = \alpha^{n+1} - \beta^{n+1} = (\alpha+\beta)(\alpha^n - \beta^n) - \alpha\beta(\alpha^{n-1} - \beta^{n-1})$

$\therefore a_{n+1} = 6a_n + 2a_{n-1}$

$\Rightarrow \therefore \frac{a_{n+1}}{2a_n} = \frac{6}{2} + \frac{a_{n-1}}{a_n} \qquad \Rightarrow \therefore \frac{a_{n+1}}{2a_n} - \frac{a_{n-1}}{a_n} = 3$

$\Rightarrow \frac{a_{10} - 2a_8}{2a_9} = 3$

Aliter

$\frac{a_{10}-2a_8}{2a_9} = \frac{(\alpha^{10}-\beta^{10}) - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2-2) - \beta^8(\beta^2-2)}{2(\alpha^9-\beta^9)} = \frac{6(\alpha^9-\beta^9)}{2(\alpha^9-\beta^9)} = 3$

since $x^2 - 2 = 6x$

D. from (ii) $\ln y = \frac{\ln x . \ln 3}{\ln 2}$

Now in (i) $\ln 2(\ln 2 + \ln x) = \ln 3(\ln 3 + \ln y)$

$\ln^3 2 + \ln^2 2 \ln x = \ln^2 3 \ln 2 + \ln x . \ln^2 3$

$= \ln x(\ln^2 3 - \ln^2 2) = -\ln 2(\ln^2 3 - \ln^2 2)$

$\ln x = -\ln 2 \Rightarrow \qquad x = \frac{1}{2}$